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Higher-twist contributions to the Structure Functions coming from 4-fermion operators*

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We evaluate the contribution of a class of higher–twist operators to the lowest moment of the Structure Functions, by computing appropriate matrix elements of six four–fermion operators in the quenched approximation. Their perturbative renormalization constants and mixing coefficients are calculated in the 't Hooft–Veltman scheme of dimensional regularization, using codes written in the algebraic manipulation computer language FORM.

1. INTRODUCTION

By performing analytic calculations and numerical simulations with quenched Wilson fermions, we have computed the contributions of some classes of higher–twist operators to moments of the structure functions.

These moments are related via an OPE to hadron matrix elements of local operators, whose twist is the difference between their dimensions and their spin. Twist–2 operators give the leading contributions, while twist–4 operators correspond to the $1/Q^2$ power corrections:

$$\begin{aligned} M_n(Q^2) &= \int_0^1 \mathrm{dx}\, \mathrm{x}^{n-2} F_2(\mathrm{x},Q^2) \\ &= C_n^{(2)}(Q^2/\mu^2, g(\mu))\, A_n^{(2)}(\mu) \\ &+ C_n^{(4)}(Q^2/\mu^2, g(\mu))\, \frac{A_n^{(4)}(\mu)}{Q^2} + O\left(\frac{1}{(Q^2)^2}\right). \end{aligned} \quad (1)$$

The leading twist contribution can be written as $M_n^{(2)} = \sum_f Q_f^2 \langle \mathrm{x}_f^{n-1} \rangle$, where Q_f is the charge of the quark of flavor f. At twist–4 level we consider the $I = 2$ pion structure function [1]

$$F_2^{I=2} = F_2^{\pi^+} + F_2^{\pi^-} - 2F_2^{\pi^0}, \quad (2)$$

which belongs to a flavor 27–plet, receives contributions only from 4–fermion operators and therefore cannot mix with twist–2 operators [2]. While the Wilson coefficients $C_n^{(k)}$ can be calculated in perturbative QCD [4,5], the computation of the matrix elements $A_n^{(k)}$ is a strong interaction problem that cannot be treated perturbatively. Some results have been obtained in the past using the MIT bag model [4], but a calculation in a reliable model–independent way and from first principles is only possible in lattice QCD.

The leading–twist contribution to the lowest unpolarized moment is given by the operator

$$O_{\mu_1\mu_2} = \frac{\mathrm{i}}{2}\bar{\psi}\gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \psi - \text{traces}, \quad (3)$$

*Talk given by S. Capitani at Lattice 99, Pisa (Italy).

[2]When mixing with operators of leading twist is forbidden, one gets rid of renormalon ambiguities completely. When such a mixing occurs, a non–perturbative computation of the Wilson coefficients in the way proposed in [2,3] would avoid renormalon problems. That method will be a useful complement to the present one for an understanding of higher–twist effects. So far however only 2–quark states have been simulated.

with Wilson coefficient $C_2^{(2)} = 1 + O(g^2)$ and forward matrix elements $\langle\vec{p}|O_{\{\mu_1\mu_2\}}|\vec{p}\rangle = 2A_2^{(2)}[p_{\mu_1}p_{\mu_2} - \text{traces}]$. The twist–4 contribution involves various operators [3], but only 4–fermion operators can contribute to the $I = 2$ combination (2), and the only 4–fermion operator that appears in (1) at order g^2 is

$$A^c_{\mu_1\mu_2} = \bar\psi\gamma_{\mu_1}\gamma_5 t^a\psi\ \bar\psi\gamma_{\mu_2}\gamma_5 t^a\psi - \text{traces}, \qquad (4)$$

with Wilson coefficient $C_2^{(4)} = g^2(1 + O(g^2))$ and forward matrix elements $\langle\vec{p}|A^c_{\{\mu_1\mu_2\}}|\vec{p}\rangle = 2A_2^{(4)}[p_{\mu_1}p_{\mu_2} - \text{traces}]$. This operator mixes under renormalization, and on the lattice there is even more freedom to mix, given the lower symmetry of the theory compared to the continuum.

2. PERTURBATIVE RENORMALIZATION

We have computed pion matrix elements of six 4–fermion operators and perturbatively calculated their renormalization factors, which relate the lattice numbers to physical quantities in the $\overline{\text{MS}}$ scheme, using the 't Hooft–Veltman prescription for γ_5. We consider the following operators in Euclidean space, symmetrized in μ and ν:

$$V^c_{\mu\nu} = \bar\psi\gamma_\mu t^a\psi\ \bar\psi\gamma_\nu t^a\psi - \text{traces} \qquad (5)$$
$$A^c_{\mu\nu} = \bar\psi\gamma_\mu\gamma_5 t^a\psi\ \bar\psi\gamma_\nu\gamma_5 t^a\psi - \text{traces} \qquad (6)$$
$$T^c_{\mu\nu} = \bar\psi\sigma_{\mu\rho} t^a\psi\ \bar\psi\sigma_{\rho\nu} t^a\psi - \text{traces} \qquad (7)$$
$$V_{\mu\nu} = \bar\psi\gamma_\mu\psi\ \bar\psi\gamma_\nu\psi - \text{traces} \qquad (8)$$
$$A_{\mu\nu} = \bar\psi\gamma_\mu\gamma_5\psi\ \bar\psi\gamma_\nu\gamma_5\psi - \text{traces} \qquad (9)$$
$$T_{\mu\nu} = \bar\psi\sigma_{\mu\rho}\psi\ \bar\psi\sigma_{\rho\nu}\psi - \text{traces}. \qquad (10)$$

Not all above operators are present in the OPE (1), they appear however when one computes the lattice radiative corrections [4].

One needs the renormalization factors of the operators on the lattice as well as in the continuum, in order to obtain physical continuum matrix elements from the Monte Carlo simulations. The connection between the two is

$$\langle O_i^{\text{cont}}\rangle = \sum_j \left(\delta_{ij} - \frac{g^2}{16\pi^2}\Delta R_{ij}\right)\cdot\langle O_j^{\text{lat}}\rangle, \qquad (11)$$

where

$$\langle O_i^{\text{cont,lat}}\rangle = \sum_j \left(\delta_{ij} + \frac{g^2}{16\pi^2}R_{ij}^{\text{cont,lat}}\right)\cdot\langle O_j^{\text{tree}}\rangle \qquad (12)$$

are the renormalized (in our case in the $\overline{\text{MS}}$ scheme) continuum and unrenormalized lattice 1–loop expressions. We computed the matrix elements on quark states, in a general covariant gauge. The differences $\Delta R_{ij} = R_{ij}^{\text{lat}} - R_{ij}^{\text{cont}}$ enter then in the renormalization factors

$$Z_{ij}(a\mu, g) = \delta_{ij} - \frac{g^2}{16\pi^2}\Delta R_{ij}, \qquad (13)$$

which convert from the lattice to the continuum, and are the object of our computations.

To evaluate the Feynman diagrams and obtain the algebraic expressions for the renormalization factors, we run computer codes written in the symbolic manipulation language FORM. To properly deal with the γ_5 matrices we have also written additional computer routines which are able to perform computations with the 't Hooft–Veltman prescription. This is the only one proven to be consistent for γ_5, and if we were not using it we would not find the right mixing factors between axial, vector and tensor operators.

Implementing the 't Hooft–Veltman scheme in a computer program is a challenging task, compounded by the fact that we are dealing here with lattice calculations. In fact, d–dimensional sums are split in 4 and $(d-4)$ dimensions, in which the Dirac algebra is different from usual (see for example [7]). The treatment of γ–matrices, already complicated by the non–validity of the summation convention on the lattice (as Lorentz invariance is broken), is then subject to much more complicated rules. When running the codes, the sum splitting increases the number of terms by about one order of magnitude, and this slows down considerably the computations, besides generating problems in memory management as well. We have found the computations presented here, which only concern Wilson fermions without any improvement, to be already very demanding.

[3] For discussions of twist–4 operator bases and general higher–twist effects, see also [5,2,6].

[4] In principle also gauge–variant operators could appear in the mixing, but there are no such 4–fermion operators with dimension 6, and 2-fermion operators do not contribute.

We use the Kawai method [8] in dealing with divergences, which allows a strong check in the analytic lattice calculations by using two different intermediate regularizations: dimensional regularization with the 't Hooft–Veltman prescription on the lattice, and an infrared–mass regularization.

Using the 't Hooft–Veltman prescription for the γ_5 matrices, the one–loop renormalized operators in the $\overline{\rm MS}$ scheme can be written in terms of the lattice operators (for $\mu = 1/a$) as [5]

$$\begin{aligned}
(V^c)^R &= V^c - g^2\Big(0.281578\ V^c + 0.000532\ A \\
&\quad +0.000997\ A^c + 0.022899\ T^c\Big) \\
V^R &= V - g^2\Big(0.348170\ V + 0.002393\ A^c\Big) \\
(A^c)^R &= A^c - g^2\Big(0.291756\ A^c + 0.000532\ V \\
&\quad +0.000997\ V^c + 0.006785\ T \\
&\quad +0.012722\ T^c\Big) \\
A^R &= A - g^2\Big(0.266750\ A + 0.002393\ V^c \\
&\quad +0.030533\ T^c\Big). \qquad (14)
\end{aligned}$$

The complete results for general N_c are given in [10]. Although used here for the pion, they are of more general application, and we will use them also for the nucleon case.

3. RESULTS

We made simulations on a $16^3 \times 32$ lattice at quenched $\beta = 6.0$, on a Quadrics QH2 at DESY Zeuthen, with a statistics of 400 configurations. To extract the expectation values of the operators we computed ratios of three–point to two–point functions of the pion, $R_O(t,\tau) = \langle\pi(t)O(\tau)\pi(0)\rangle/\langle\pi(t)\pi(0)\rangle$.

After extrapolating the matrix elements for $F_2^{I=2}$ to the chiral limit and using the renormalization factors (14) at $Q^2 = \mu^2 = a^{-2}$, we obtain $A_2^{(4)} = 0.085(27) f_\pi^2$, which multiplied by $C_2^{(4)}$ and the kinematical factor gives the twist–4 contribution to the lowest moment of $F_2^{I=2}$:

$$M_2^{(4)} = 0.085(27)\frac{f_\pi^2 g^2}{Q^2} + O(g^4). \qquad (15)$$

For the leading contribution we have $\langle \mathrm{x}_i \rangle = 0.273(12)$ (independent of flavor), so that weighting with the charges, say of the π^+, one gets [11]

$$M_2^{(2)\pi^+} = 0.152(7) + O(g^2). \qquad (16)$$

The relative magnitude of the twist–4 and twist–2 contributions to the lowest moment of the pion structure functions turns out to be small, but is expected to be larger for the nucleon, where the scale f_π is replaced by m_N. We are now studying the nucleon with the same machinery and using the same renormalization factors as here.

[5] For a similar calculation in the context of weak matrix elements, involving a different set of operators and a different scheme, see also [9].

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